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Fluid Dynamical modeling: Clinical-Practical Aspects

Vincenzo Coscia

Department of Mathematics and Computer Science
and
Research Center "Mathematics for Technology, Medicine & Biosciences"

University of Ferrara

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Outline:

1. Brief on modeling in vascular biomechanics

2. Report on modeling of the saphenofemoral junction

3. Results on optimization of the saphenofemoral venous district

4. Perspectives

Mathematics-Engineering workgroup:
N. Cavallini, A. Benfenati (Post-Doc)
D. Bellandi (PhD Student)

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What is a hemodynamic model?

A hemodinamic model is a computer-assisted reconstruction of a prototypical clinical situation

A HM is made up by:

1) A *mathematical framework*, consisting of the equations describing the physics of the problem and the numerical techniques used to solve them. Typically, the choice depends on a balance between the desired accuracy and the computational cost

2) A *geometric setting*. A detailed mathematical scheme needs an accurate geometric settings, obtained upon an average procedure based, in our case, on 50 patients morphologic records

3) Suitable *flow conditions*, mainly corresponding to inflow-outflow boundary data, measured *in vivo*

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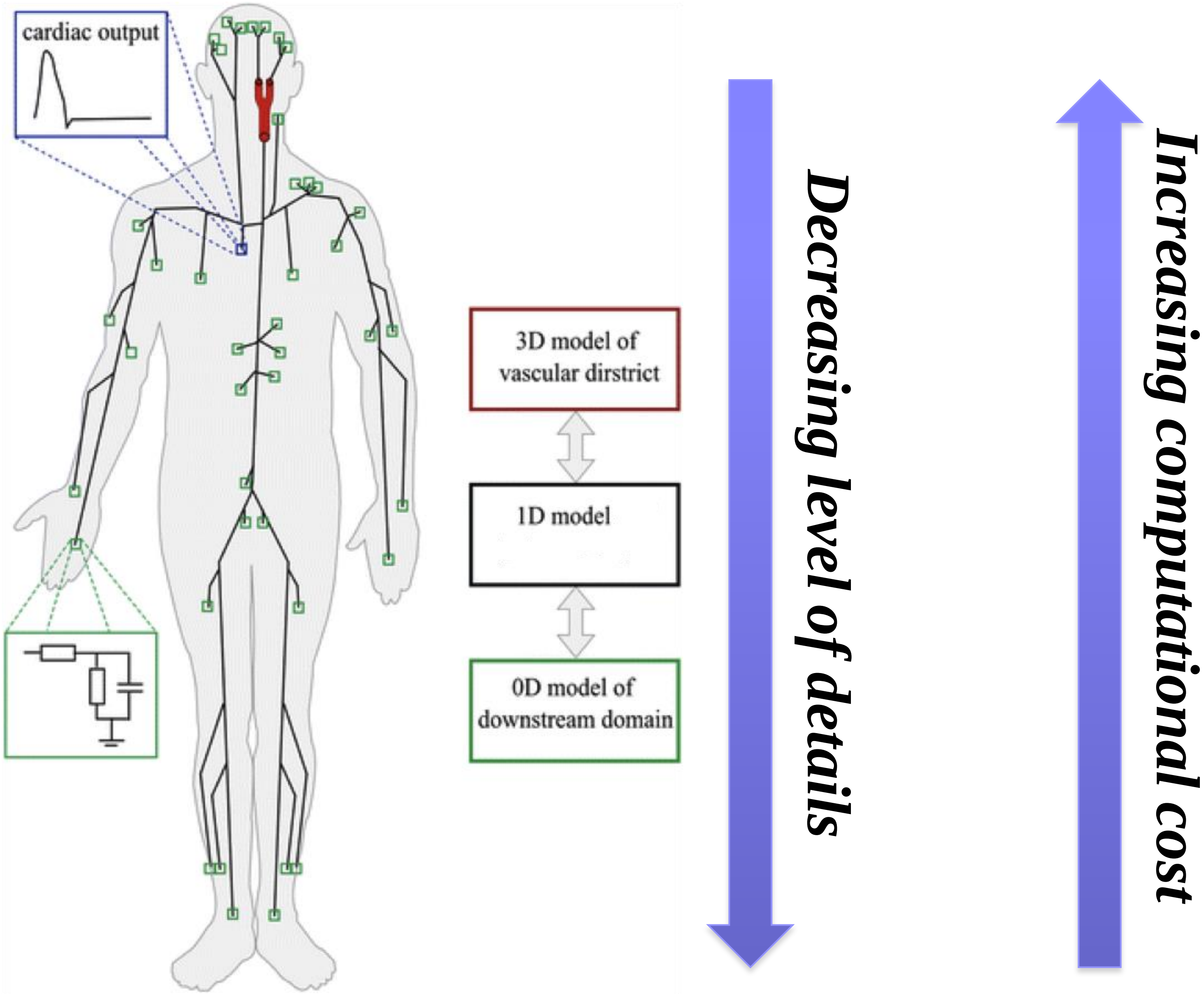
Multiscale modeling of vascular system



The level of details is low on the downstream domain, is high on the local portion of the vascular system

Mathematical framework

The right choice of the mathematics used to describe (part of) the vascular system depends on the required level of detail

Since we are interested in the blood dynamics of a delimited region (the saphenofemoral junction) for which we wish an accurate description, we are led to use the 3D Navier-Stokes system, implemented by a 3D finite-elements numerical scheme running on a workstation DELL P M4300

$$\begin{cases} \rho\left(\dfrac{\partial \mathbf{u}}{\partial t} + \mathbf{u}\cdot\nabla\mathbf{u}\right) = \mathbf{F} - \nabla p + \mu\Delta\mathbf{u} \\ \mathrm{div}\,\mathbf{u} = 0 \qquad \Omega\,, t > 0 \end{cases}$$

$$\mathbf{u}(\mathbf{x},0) = \mathbf{u}_0(\mathbf{x}) \qquad \text{in } \Omega,\ t = 0$$

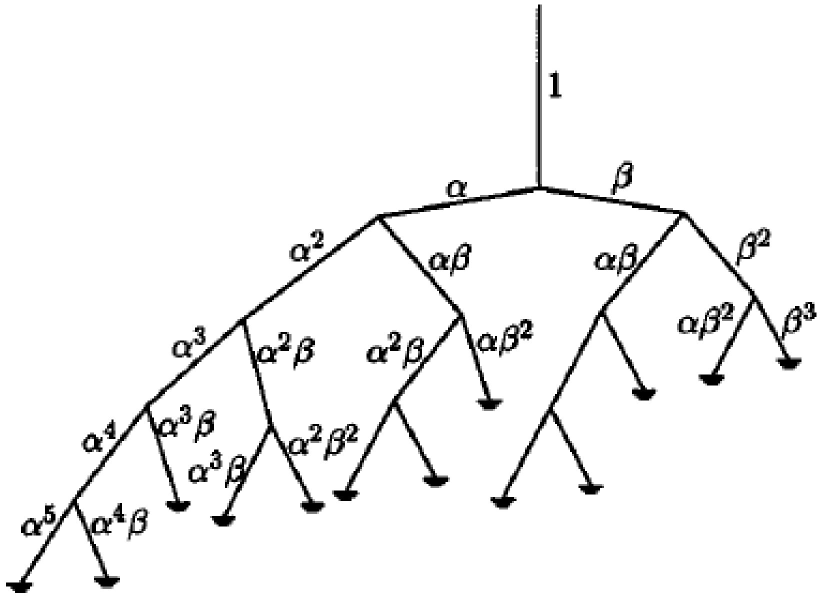
$$\mathbf{u} = \mathbf{g} \qquad \text{on } S_i\,,\ t > 0$$

$$-p\,\mathbf{n} + \mu\nabla\mathbf{u}\cdot\mathbf{n} = \bar{p}\,\mathbf{n} \qquad \text{on } S_o\,,\ t > 0$$

3D Finite Elements numerical code

Our approach:

• in the large vessels, blood flux and pressure are predicted at any point (1D)
• in the small vessels, a relation between flux and pressure yields outflow boundary conditions for the large vessels



1: parental vessel, a, b scale factors

Bifurcation conditions: $q_{\mathrm{pa}} = q_{d_1} + q_{d_2}$. $p_{\mathrm{pa}} = p_{d_1} = p_{d_2}$.

Equations for large vessels:

$$\frac{\partial q}{\partial x} + \frac{\partial A}{\partial t} = 0. \qquad (6)$$

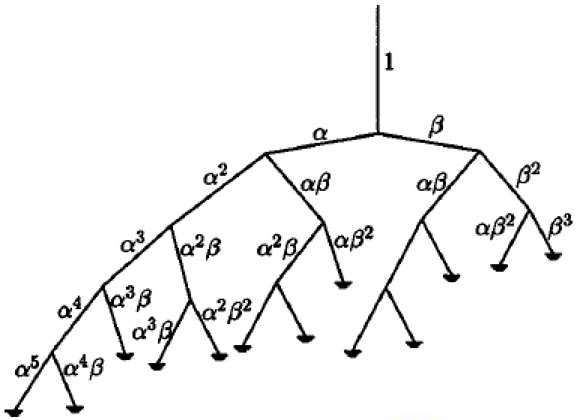
$$\frac{\partial q}{\partial t} + \frac{\partial}{\partial x}\left(\frac{q^2}{A} + B\right) = -\frac{2\pi\nu q R}{\delta A} + \frac{\partial B}{\partial r_0}\frac{dr_0}{dx}. \qquad (13)$$

$$p(x,t) - p_0 = \frac{4}{3}\frac{Eh}{r_0}\left(1 - \sqrt{\frac{A_0}{A}}\right), \qquad (14)$$

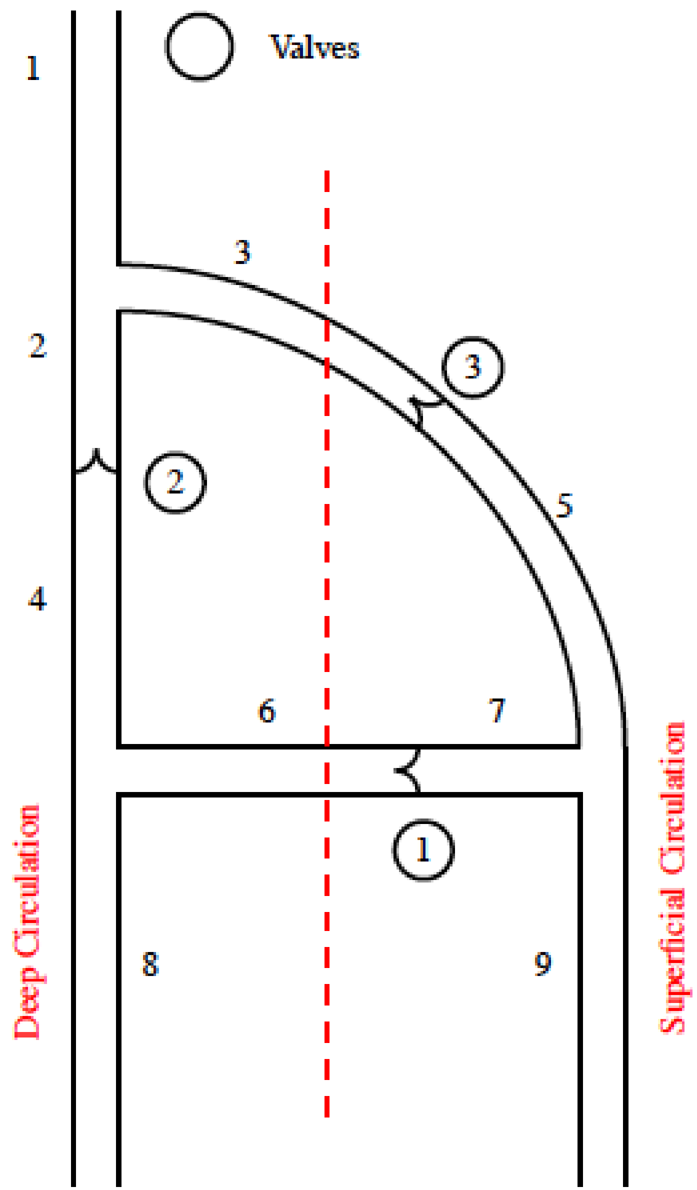
- Inflow boundary condition (by MR)
- Bifurcation conditions

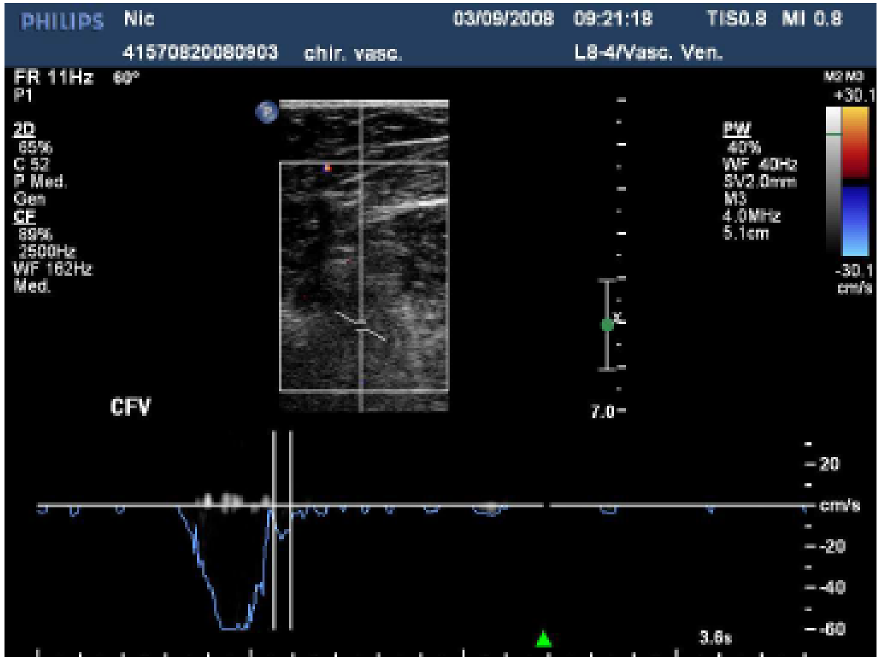
$$q_{\mathrm{pa}} = q_{d_1} + q_{d_2}. \qquad p_{\mathrm{pa}} = p_{d_1} = p_{d_2}.$$

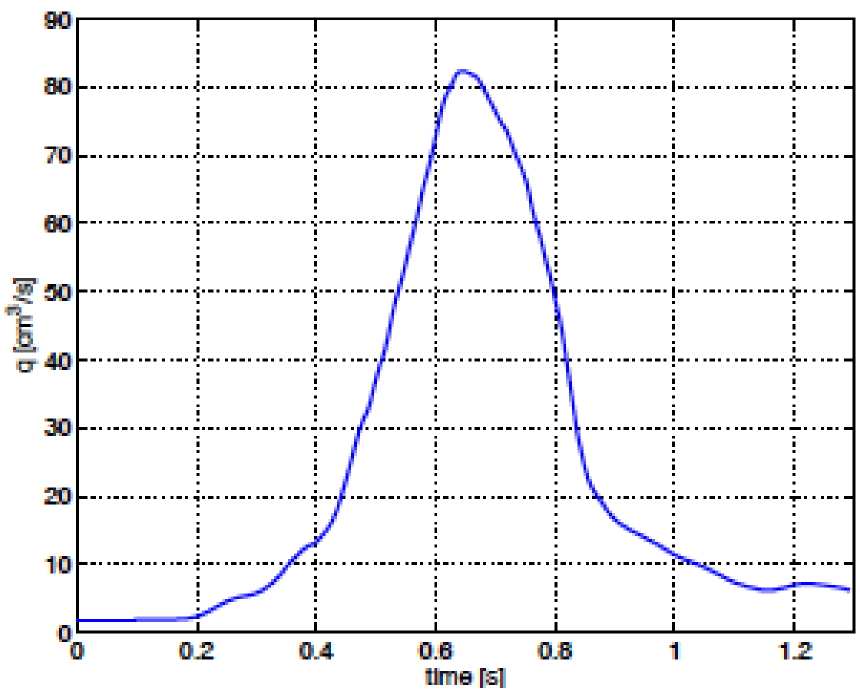
- Outflow boundary conditions



A benchmark application: Malfunction of a saphenous valve







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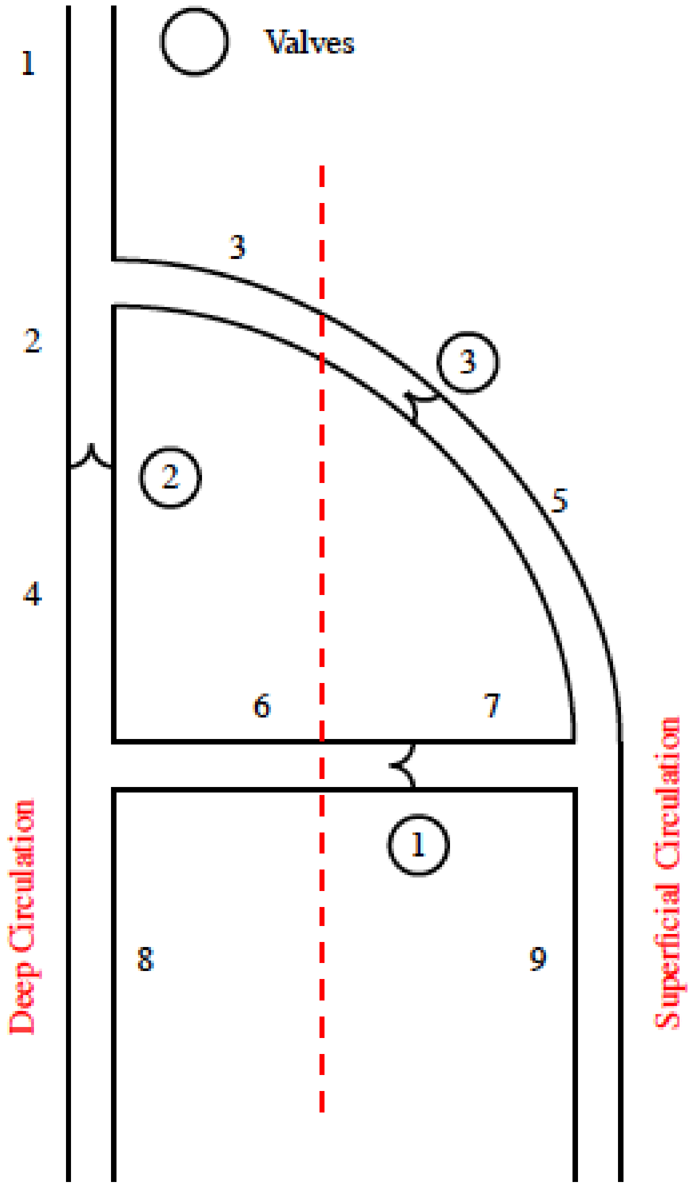
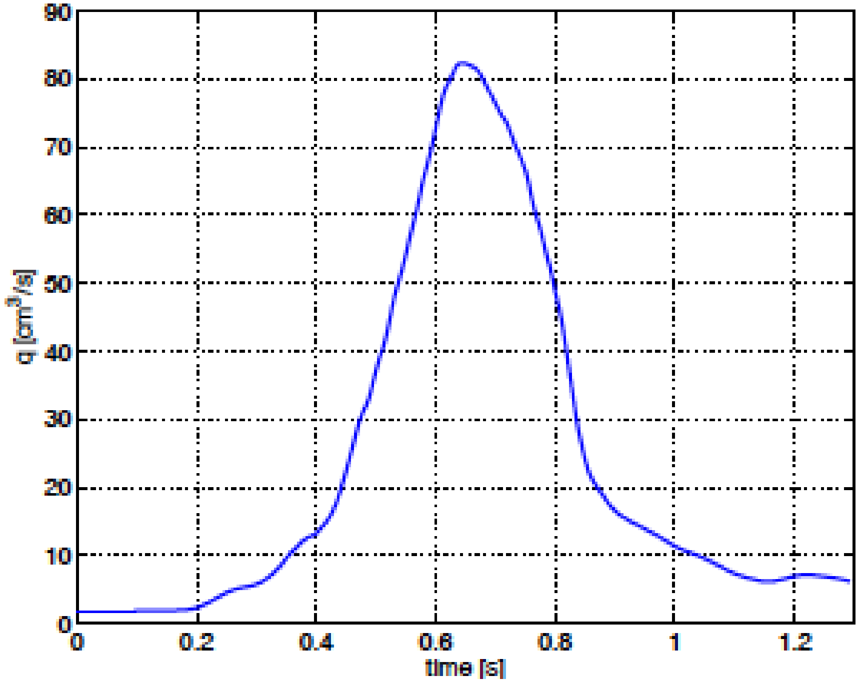
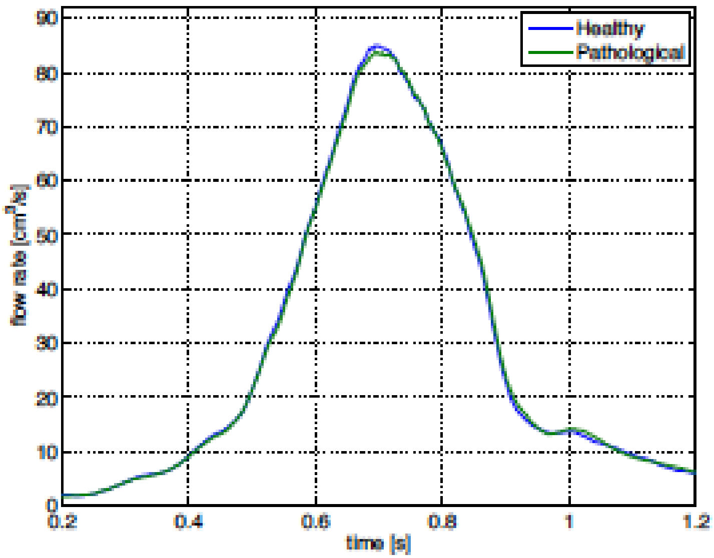
A benchmark application: Malfunction of a saphenous valve

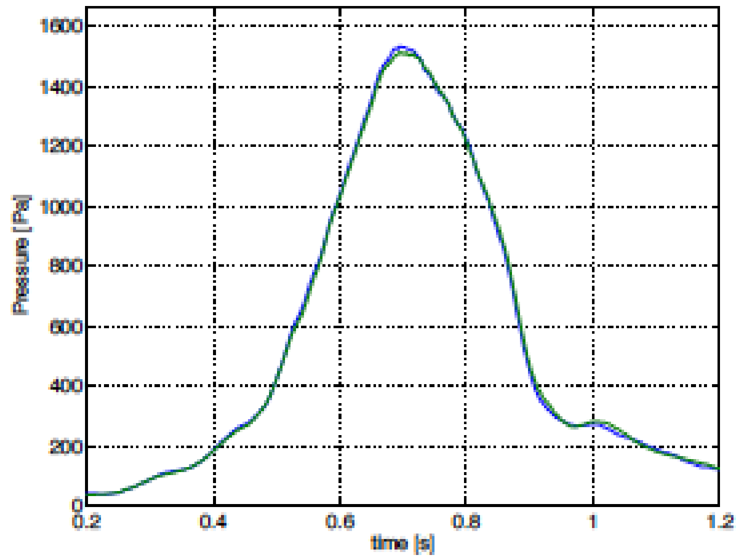


Vessel Name	Length [cm]	Diameter [cm]	Vessel Number
Common Femoral Vein	4.00	1.580	1
Common Femoral Vein	7.50	1.400	2 - 4
Grand Saphenous Vein	8.00	0.729	3 - 5
Dodd	1.20	0.127	6 - 7
Grand Saphenous Vein	5.00	0.472	9
Common Femoral Vein	5.00	1.400	4

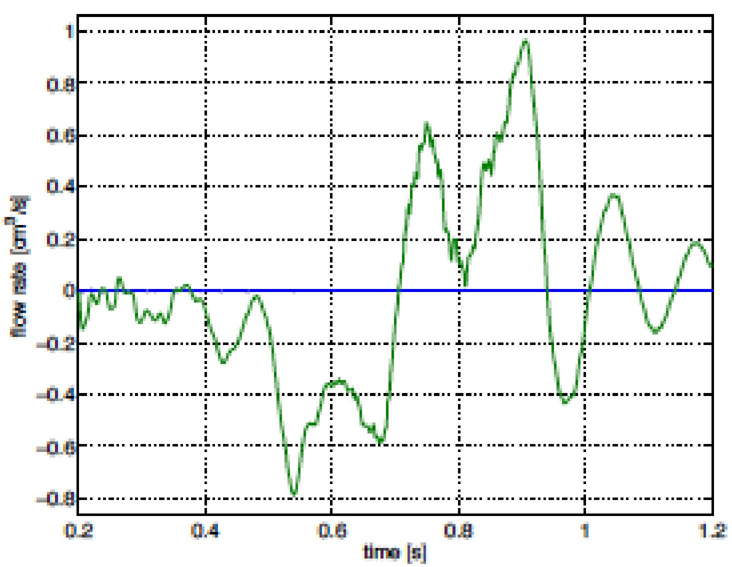




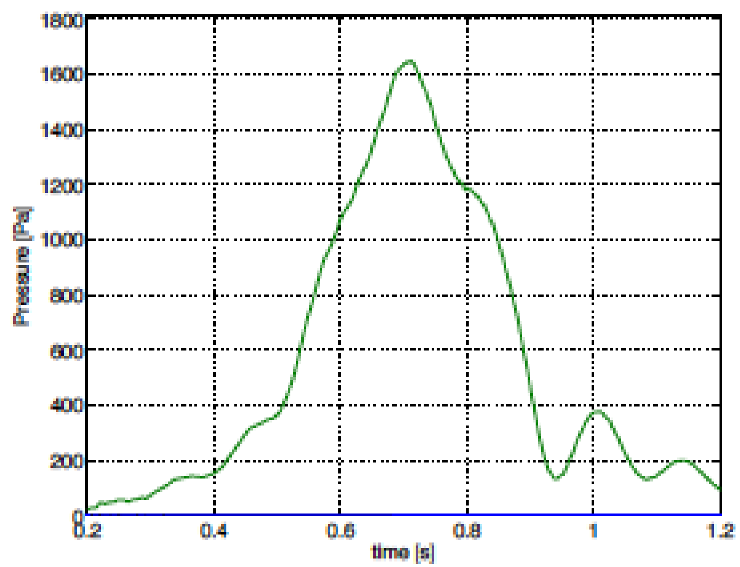
(a) Flow rate at midpoint of vessel 1, femoral vein.



(b) Pressure at midpoint of vessel 1, femoral vein.



(e) Flow rate at midpoint of vessel 5, saphenous vein.



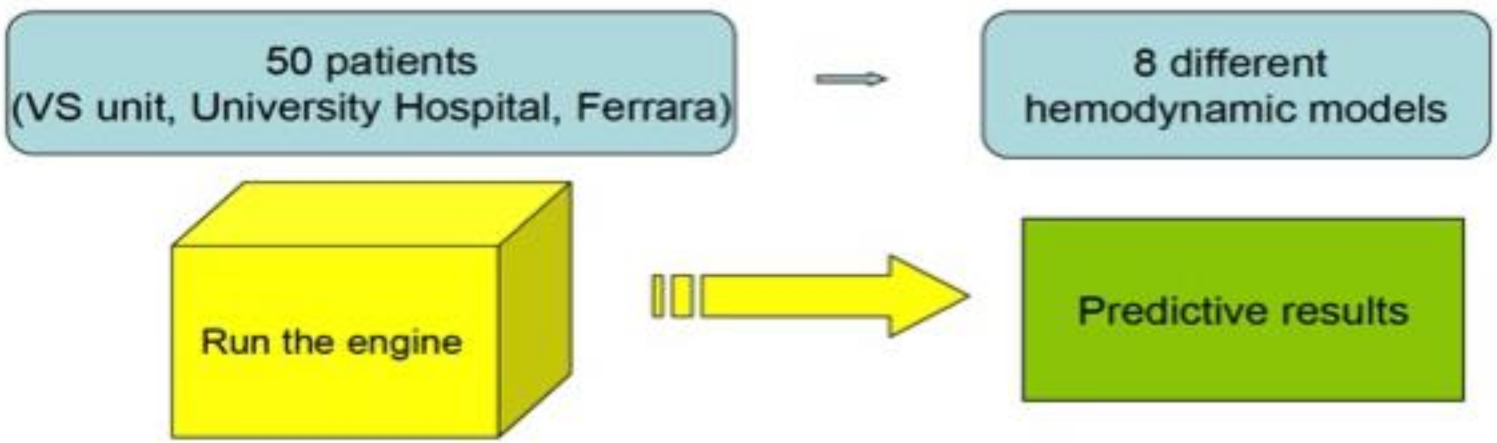
(f) Pressure at midpoint of vessel 5, saphenous vein.

A second application: Varying the geometrical setting of saphenofemoral junction district

Report on a multidisciplinary approach to the hemodynamics of the saphenofemoral junction

Materials and Methods:

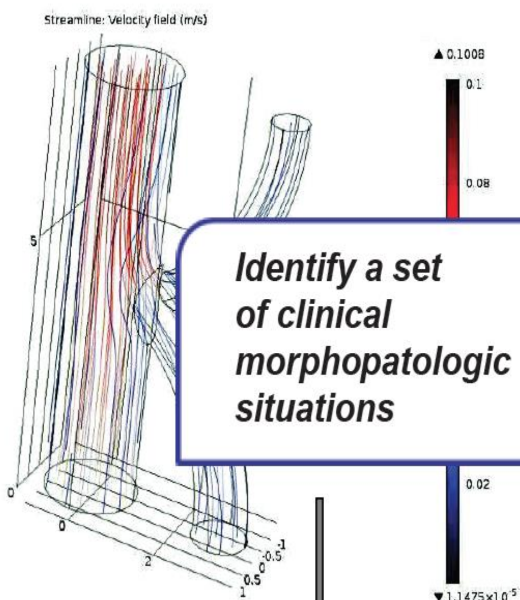
Coupling of ECD hemodynamic measures and numerical reconstruction of local morphologic fluid dynamical parameters



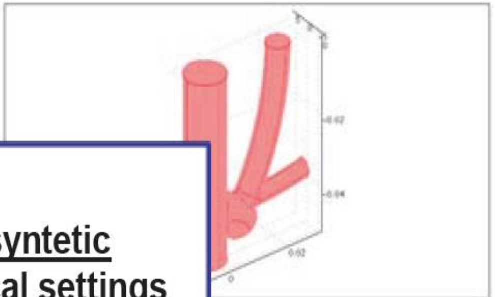
Geometrical settings

50 phlebologic patients

Measure of morphologic and flow data

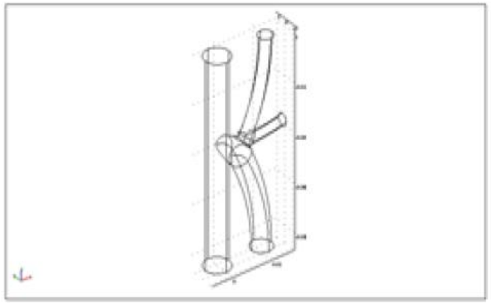


Identify a set of clinical morphopatologic situations

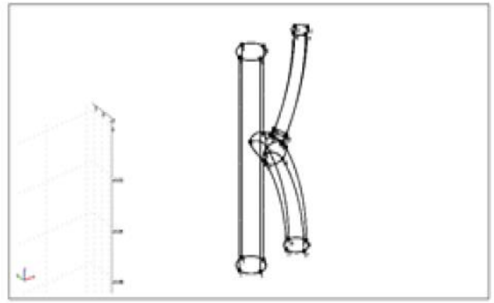


Produce syntetic geometrical settings

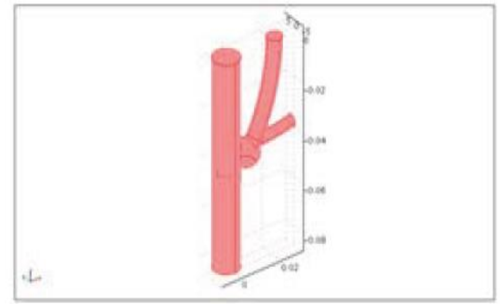
7 classes of geometric settings are identified (no. of patients in parenthesis)



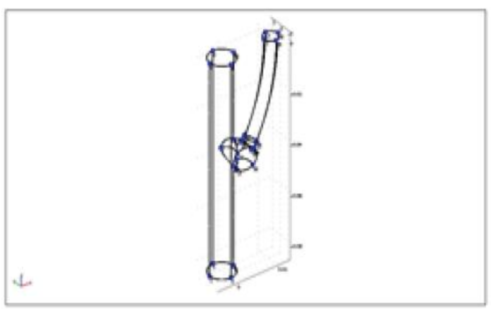
Complete SF junction (8)



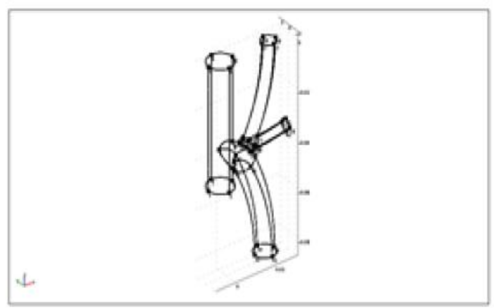
Collateral missing (6)



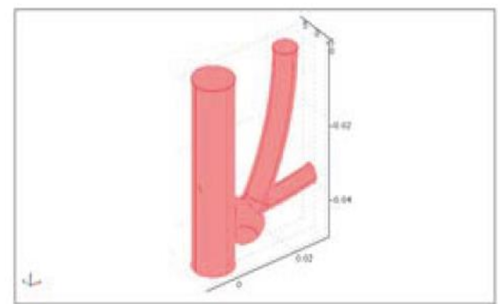
Saphenectomy, collateral present (8)



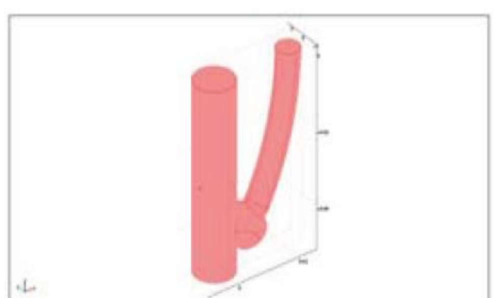
Saphenectomy, collateral missing



Continent femoral valve (6)



Saphenectomy, continent femoral valve (7)



Saphenectomy, continent femoral valve, collateral missing (8)

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Model 1		Model 2	
Geometrical setting	1	*Geometrical setting*	2
Systolic velocity data.	physiologic	*Systolic velocity data.*	physiologic

Model 3		Model 4	
Geometrical setting	3	*Geometrical setting*	4
Systolic velocity data	physiologic	*Systolic velocity data*	physiologic

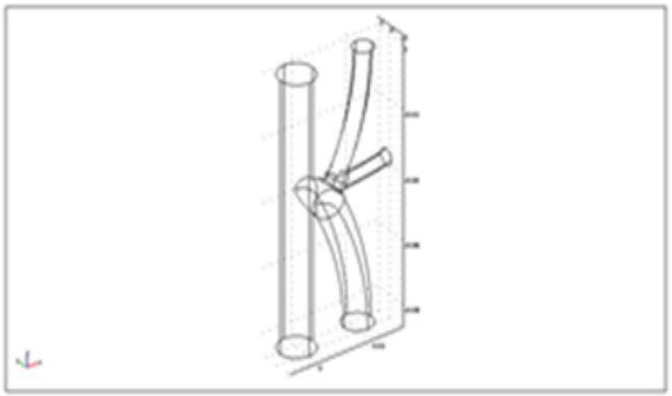
Model 5		Model 6	
Geometrical setting	1	*Geometrical setting*	5
Diastolic velocity data.	Reflux on the femoral vein Incontinent femoral valve Incontinent ostial valve	*Diastolic velocity data*	Reflux on the femoral vein Continent femoral valve Incontinent ostial valve

Model 7		Model 8	
Geometrical setting	6	*Geometrical setting*	7
Diastolic velocity data	Reflux on the femoral vein Continent femoral valve Incontinent ostial valve	*Diastolic velocity data*	Reflux on the femoral vein Continent femoral valve Incontinent ostial valve .

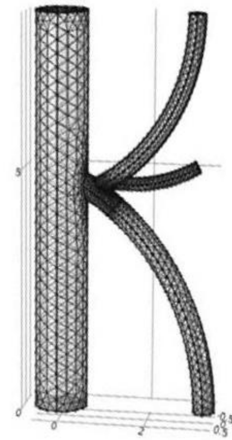
In order to achieve the full hemodynamic model, flow conditions (boundary data) must be implemented on the geometrical settings

For all the settings, the pressure values obtained assuming hydrodynamic flow conditions (Poiseuille flow)

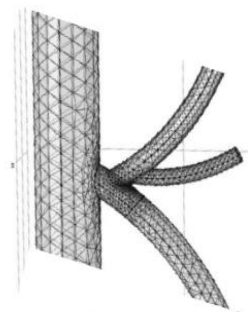
Example



Model 1

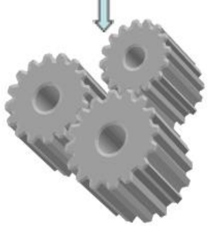


The geometric object is segmented and a mesh is created

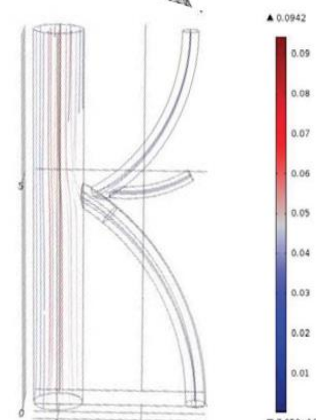


The mesh is not uniform, as it must be finer close to the junction to get better results

The equations of hydrodynamics (Navier-Stokes) are implemented in the described geometries, under the prescribed flow conditions

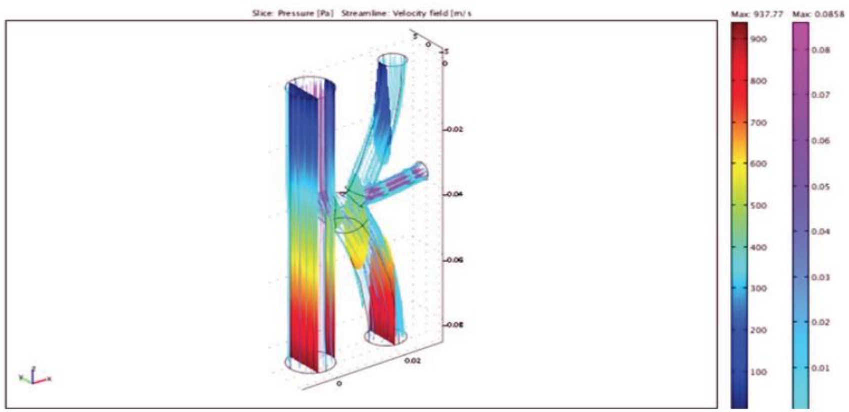


Let the machinery work

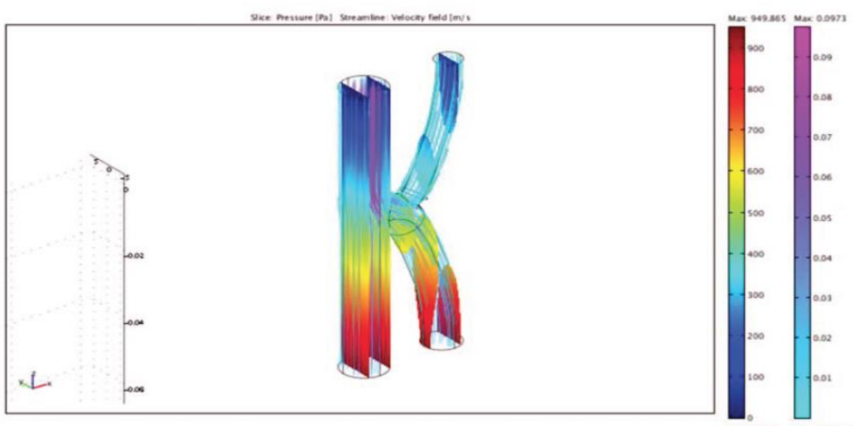


Get the results

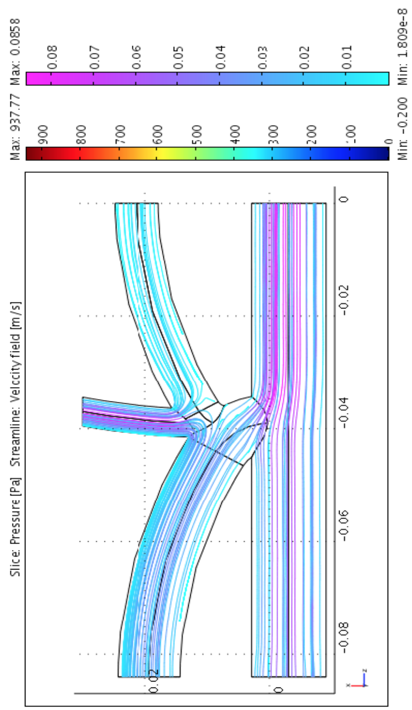
Results

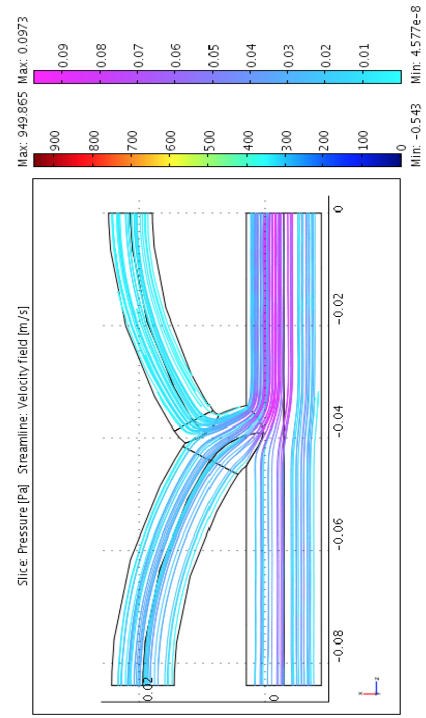


Model 1. In this case the physiologic systolic flow conditions are reproduced. The hydrostatic column on the epigastric vein allows the blood to flow down into the femoral, and the higher pressure from the saphena and from the femoral leads the blood to the deep circulation. It is worth observing the «escape» effect from the collateral branch draining from the whole system

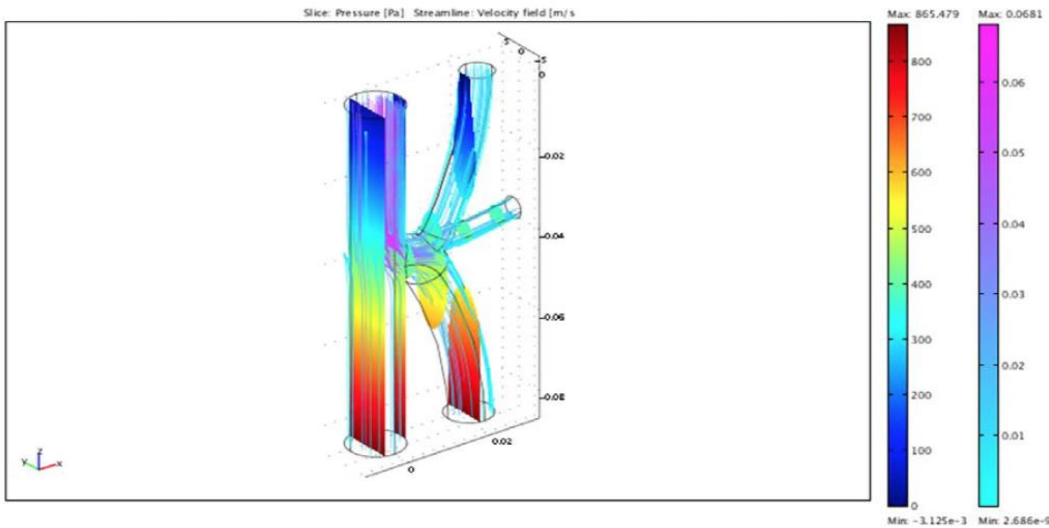


Model 2. As in Model 1, when the collateral branch Is missing

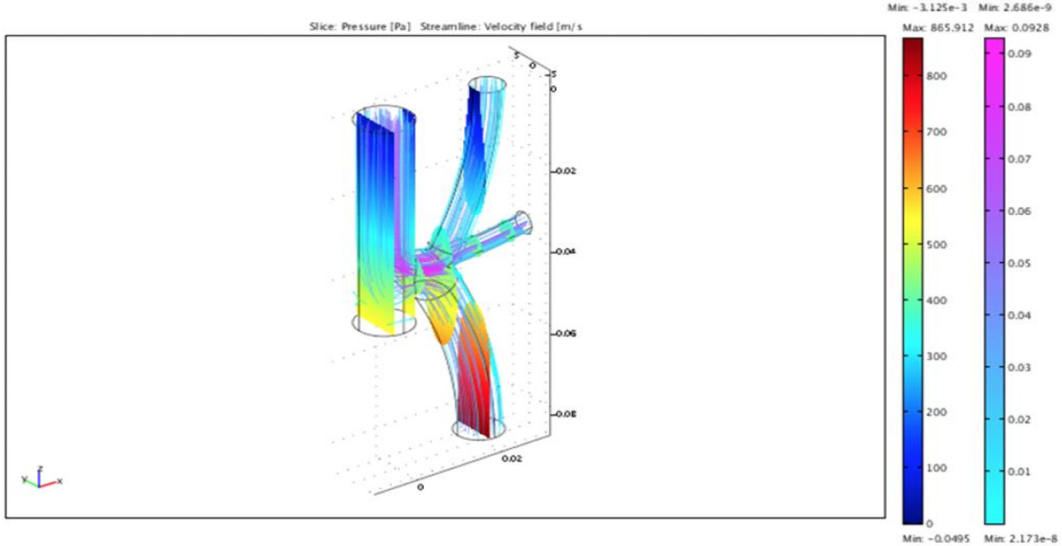




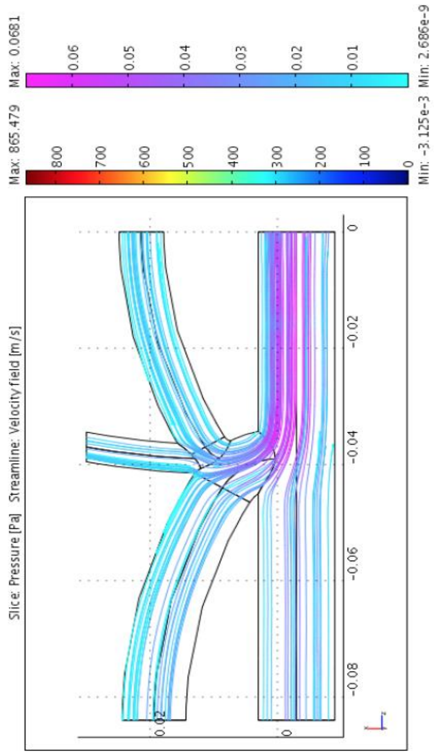
Comparison between Models 1 and 2: Both models, solved under physiologic systolic flow conditions, show the flowing down from the epigastric branch under the action of hydrostatic pressure, with the flow physiologically oriented from the superficial to the deep circulation. In case (1) it is possible to observe how the presence of the collateral branch represents an escape direction for the blood flow, causing a draining effect and an hypertension of the superficial tree.

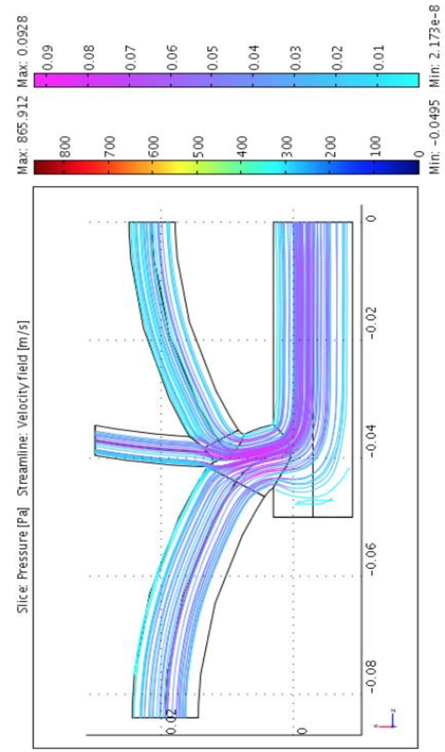


Model 5. Valsalva maneuver with refluent femoral vein toward superficial circulation, due to incontinent ostial and saphenofemoral valves



Model 6. Valsalva maneuver with continent sub-ostial valve and incontinent ostial one. In this situation, the whole flow is deviated in direction of superficial.





Comparison of Models 5 and 6. When the femoral vein is refluent, the simulations show how the continence of the sub-ostial femoral valve causes a hypertension of the superficial

A third application: Global optimization of the venous tree: max flow-min reflux

Translate the venous network as a graph $G(N, A)$, evaluate the maximum flux between node s and node t as:

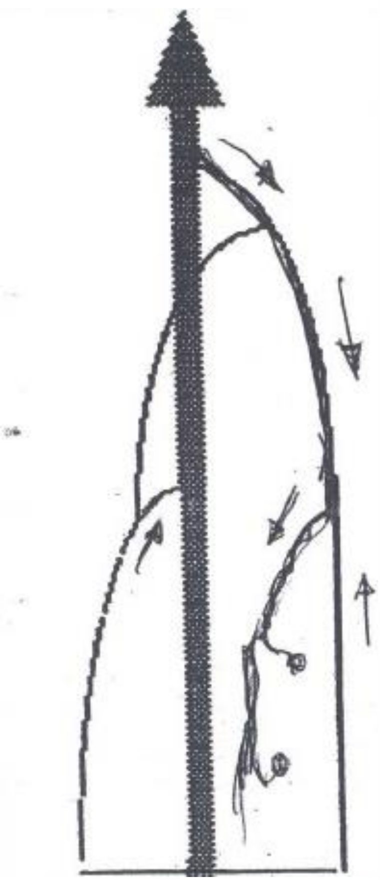
$$max \sum_{(s,j)\in A} x_{sj} \quad \text{such that} \quad \sum_{(i,j)\in FS(i)} x_{ij} - \sum_{(j,i)\in BS(i)} x_{ji} = 0 \quad \forall i \in N \setminus \{s,t\}$$

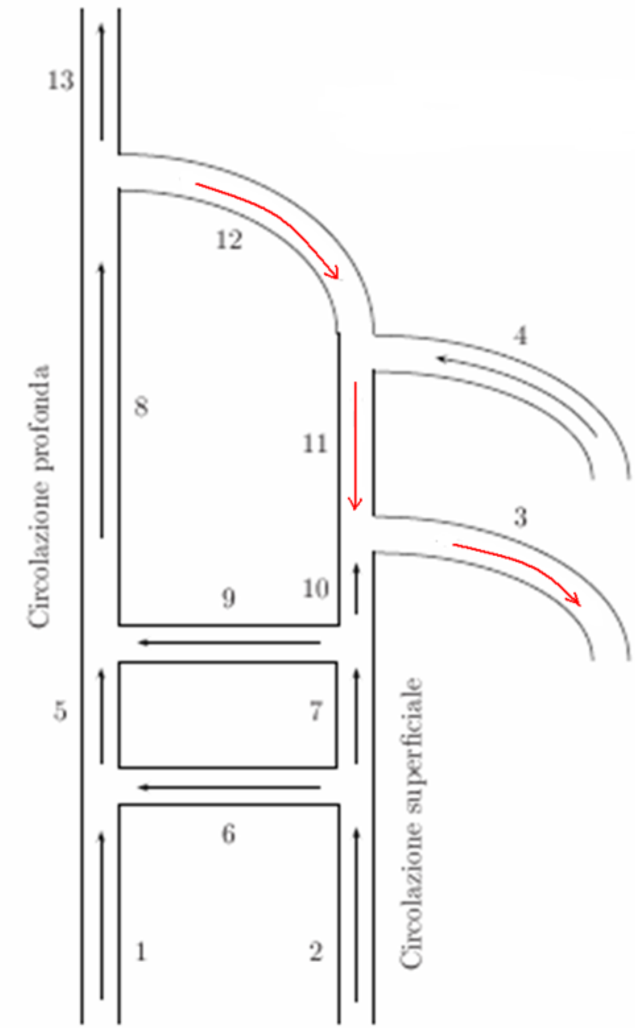
$$0 \leq x_{ij} \leq u_{ij}.$$

then, find the graph $\tilde{G}(\tilde{N}, \tilde{A})$, that differs from $G(N, A)$ in a finite $(< \nu(A))$ number of arcs that minimize the difference $|\bar{x} - x|$ in the total flux and restore (in the largest number of arcs) the direction of the flow (**increasing-decreasing paths Ford-Fulkerson algorithm**)

Clinical case (O.U. Vascular Surgery, University Hospital, Ferrara):

Female, 46 y.o., overweight, reflux in the high part of great saphenous vein, discharging in a collateral

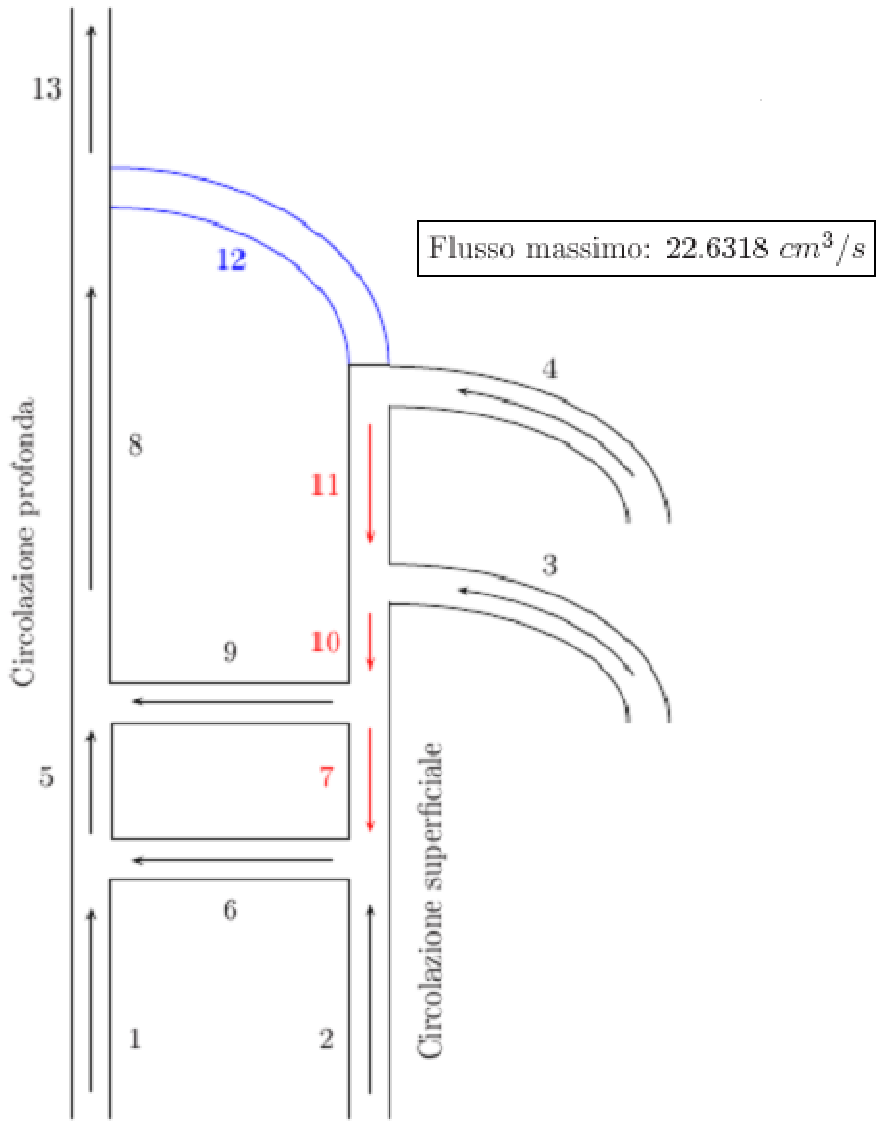




Flusso massimo: 27.4516 cm^3/s

	Vena	l (cm)	d (mm)
1	Femorale	7	12
2	Grande Safena	7	5
3	Accessoria	7	7
4	Collaterale	7	3
5	Femorale	4	10
6	Perforante n.1	2	4
7	Grande Safena	4	7
8	Femorale	13	14
9	Perforante n.2	2	2
10	Grande Safena	3	7
11	Grande Safena	5	7
12	Grande Safena	6	7
13	Femorale Comune	4	18

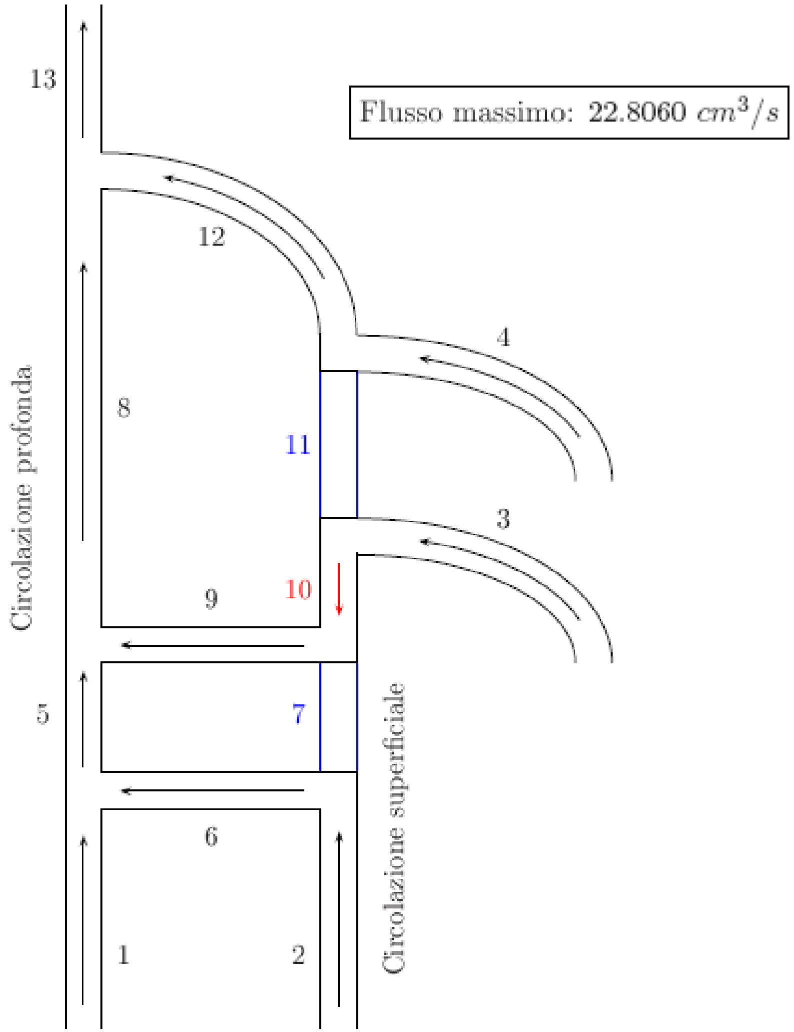
Let's force the software to ligate the "guilty" branch



	Vena	q (cm^3/s)
1	Femorale	21.8516
2	Grande Safena	0.2190
3	Accessoria	0.4792
4	Collaterale	0.0820
5	Femorale	22.5836
6	Perforante n.1	0.7320
7	Grande Safena	-0.5130
8	Femorale	22.6318
9	Perforante n.2	0.0482
10	Grande Safena	-0.5612
11	Grande Safena	-0.0820
12	Grande Safena	0
13	Femorale Comune	22.6318

there is still a high reflux in the saphena, the maximum drainage decreases

Let’s leave the algorithm to choose which branch has to be ligated



	Vena	q (cm^3/s)
1	Femorale	22.1220
2	Grande Safena	0.4616
3	Accessoria	0.0482
4	Collaterale	0.1742
5	Femorale	22.5836
6	Perforante n.1	0.4616
7	Grande Safena	0
8	Femorale	22.6318
9	Perforante n.2	0.0482
10	Grande Safena	-0.0482
11	Grande Safena	0
12	Grande Safena	0.1742
13	Femorale Comune	22.8060

only a small reflux persists in the saphena (in an open shunt), the maximum drainage is essentially the same as in the previous case

Perspectives:

The mathematical modeling in biomechanics, particularly in vascular hemodynamics, reveals, in our opinion, to be a valuable tool supporting the clinical practice.

In these slides we've reported some results concerning the modeling of the saphenofemoral junction's district, both in physiologic and in pathologic conditions.

We are presently looking to extend the modeling in order to include ascpects related to the dermis modification in phlebologic and lymphologic diseases.

Thanks